



Reading 19

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RESISTIVE, INDUCTIVE AND CAPACITIVE CIRCUITS

The purpose of this reading is to understand the combination of resistive, inductive and capacitive circuits and the concepts of impedance, quality factor or 'Q', and resonant circuits.

I am going to avoid the maths where possible, though I will be giving you some equations. The ones you need to remember I will mention. However, you will not actually have to use many of the equations in the exam.

IMPEDANCE (Z)

Impedance is the total opposition to current flow in an AC circuit.

Impedance is the opposition to current flow in an AC circuit and is measured in ohms. There is no inductive or capacitive reactance in a DC circuit. However, in an AC circuit there is resistance, inductive and capacitive reactance. All of these oppose current flow and the combined opposition to current flow is called **impedance**.

Since impedance includes capacitive and inductive reactance it is frequency dependent. Resistance is not frequency dependent.

Impedance cannot except in the case where reactances are equal and opposite be expressed as a single number. There are two common methods of expressing impedance.

1. The polar form and;
2. The rectangular form

The rectangular form is more often used in amateur radio and devices such as noise bridges and impedance meters display this form and radio amateurs are usually concerned about the amount of unwanted inductance or capacitance in antennas (resonant antennas do not have either of these)

POLAR FORM

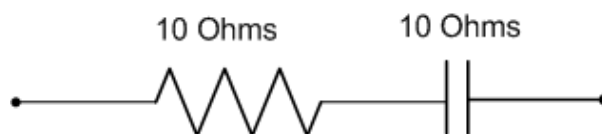


Figure 1 - a series RC circuit

The circuit of figure 1 consists of 10 ohms of resistance in series with 10 ohms of capacitive reactance. The formula for the magnitude of the total resistance is shown on the right.

For exam or assignment purposes, you do **not** use the polar method.

When impedance is expressed in Polar Form the two numbers are called Magnitude and Phase Angle. The magnitude is found by using the equation in figure 1 and the phase angle 'θ' (the amount of lead or lag of current) is found from

$\tan \theta = X/R$ or $\theta = \text{Arctan } X/R$. (Arctan means Inverse Tangent)

Let's work out the impedance of the circuit in figure "polar form". Note there is no X_L so we just use X_C .

$$Z_{mag} = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z_{mag} = \sqrt{10^2 + 10^2}$$

$$Z_{mag} = \sqrt{10^2 + 10^2}$$

$$Z_{mag} = 14.14$$

and now the phase angle:

$$\tan \theta = \frac{X_C}{R}$$

$$\tan \theta = \frac{10}{10}$$

$$\tan \theta = 1$$

$$\text{ArcTan } 1 = 45^\circ$$

So the impedance in figure 1 in polar form is said to be **14.14 Ohms / Phase angle 45 degrees**. The current leads (since it's a capacitive circuit) by 45 degrees.

When we discussed capacitive reactance, we talked about how the current **leads in a capacitive circuit**. In fact, in a capacitive only circuit the current leads the voltage by 90 degrees. Another way of saying this is, in a capacitive circuit the phase angle is 90 degrees (leading). In a circuit containing both resistance and capacitance, the phase angle will be between 0 and 90 degrees. There is **no** requirement for us to calculate the exact phase angle.

RECTANGULAR FORM

For our purposes, the impedance above is **best** described just as it is in the circuit, ie. 10 ohms of resistance in series with 10 ohms of capacitive reactance. Radio amateurs and most others use this sometimes more convenient and easier rectangular form of describing impedance. This is especially so when working with antennas

A rectangular form of writing the impedance in figure 1 is - is **10-j10** (said as: ten minus jay ten). The 'j' indicates that this part of the impedance is reactive. The **minus sign indicates**

that the reactance is capacitive. “J” alone indicates the impedance is inductive and “j0” means there is no net reactance at all

Another Example:

50-j4 is shorthand for 50 ohms of resistance in series with 4 ohms of capacitive reactance.

If the minus sign is not present then the reactance is inductive.

70j30 is shorthand for 70 ohms of resistance in series with 30 ohms of inductive reactance.

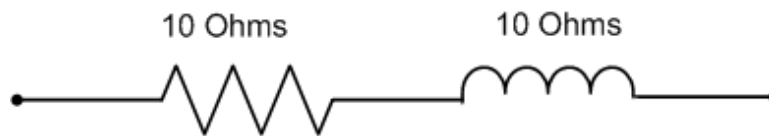


Figure 2 - a series RL circuit

In the circuit of figure 2 it is sufficient to describe this impedance as **10j10 ohms**.

It is important to remember that any circuit can be reduced to a resistance and either a single inductive or capacitive reactance. If there are two reactance the larger cancels the smaller.

A circuit cannot be inductive and capacitive at the same time. The effect of inductive reactance on current is to cause the current to lag the voltage. The effect of capacitive reactance is to cause the current to lead the voltage. Both of these effects are opposite. Therefore, the circuit will either be capacitive or inductive, but not both.

If you have trouble remembering which leads and lags, remember this:

L is for inductance - L is for Lag.

(That is, current lags the voltage in an inductive circuit)

Complicated circuits with lots of resistors, capacitors and inductors can be reduced to a single resistance and a single reactance, either capacitive or inductive.

The circuit below is a series RLC circuit. Since the inductive reactance is 100 ohms and the capacitive reactance is 50 ohms, the net reactance of the circuit is 50 ohms of inductive reactance.

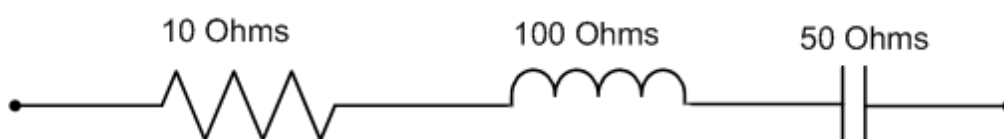


Figure 3 - a series RLC circuit

The impedance of the figure 3 circuit is **10j50 ohms**.

WHEN $X_L = X_C$

When a circuit contains exactly the same amount of inductive and capacitive reactance, the net reactance is zero and the circuit is resistive.

So if a circuit contains 100 ohms of resistance in series (or parallel) with 200 ohms of inductive reactance and 200 ohms of capacitive reactance, the impedance of the circuit is 100j0 or simply 100 ohms.

TUNED CIRCUITS OR RESONANT CIRCUITS

When inductance and capacitance are combined in a circuit so that their reactances are equal (no net reactance) the circuit is said to be **resonant**.

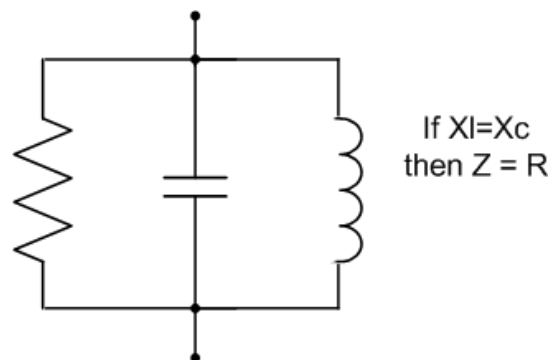


Figure 4 - a parallel RLC circuit

In the parallel circuit of figure 4, the inductive and capacitive reactances are equal so the net impedance is the value of the resistance. This circuit is also resonant if the two reactances are equal.

RESONANCE.

Resonant circuits have some very useful properties in electronics and radiocommunications, however let's talk a bit about what resonance is.

There is mechanical resonance as well as electrical resonance and both are very much the same. A taut string like that on a guitar, if plucked, will resonate at a specific frequency. The length of the string will determine on what frequency it will resonate.

A metal tuning fork is designed to resonate at a specific frequency. When you strike a tuning fork it will vibrate at the same single frequency every time, because the tuning fork is resonant at that frequency.

If you hit the rim of a wineglass with a hard object it will "ring" and it will always ring at the same frequency because it is resonant at that frequency.

In all the cases above, we had to deliver some energy to the object (guitar string, tuning fork or wineglass) to get it to resonate.

Have you ever been listening to loud music and at certain times the window may rattle, or an object on a table may start to move (vibrate with the music)? The window will start to vibrate if sound waves at its resonant frequency strike it. Since music contains many frequencies, the window will only vibrate when its resonant frequency is present in the music.

HOW DOES A SINGER BREAK A GLASS?

It is true that a singer, provided they can reach the right note, can break a wine glass. A wine glass has a frequency at which it will resonate. Making a loud noise will not break a wine glass. However, if a singer reaches and sustains a note which is equal to the resonant frequency of the wine glass, then the wine glass will absorb energy from the sound wave and it will begin to resonate (vibrate), and if the amplitude (loudness) is enough, it will shatter.

The wineglass absorbs energy on its resonant frequency. All objects, even electrical circuits, will absorb energy on their resonant frequency.

TUNING FORK EXAMPLE

If we had two tuning forks designed to create the same note (frequency) and we struck one fork to make it resonate, and then held the two tuning forks say 50 mm apart, the tuning fork that you did not strike would begin to resonate, as it would absorb sound wave energy from the vibrating tuning fork with ease, and begin to vibrate itself.

In electronics we don't say something vibrates. We say it **oscillates**. A mechanic might say the guitar string when plucked would vibrate at 800 times per second. A musician would say it makes such and such a note. An electronics person will say it oscillates at 800 Hertz.

DAMPED OSCILLATIONS

If a guitar string or tuning fork is given energy we have learnt that it will oscillate at its resonant frequency. All things being perfect, the wave produced will be the shape of a sine wave. Now what happens to the sound wave emitted from a guitar string or whatever, after the initial pulse of energy is given to it?

The guitar string will oscillate strongly at first and then slowly decrease in amplitude until it no longer makes any sound. Important - the frequency stays the same but the amplitude dies down to nothing. This is called a damped oscillation.

CONTINUOUS OSCILLATION

A swing (perhaps with a child on it though the child is not an essential ingredient) will oscillate, as it is a pendulum. We know that a pendulum will move back and forth at the same frequency (or period) every time. This is a fact because we use pendulums to create clocks. The amplitude of the swing of the pendulum will slowly damp but the frequency will remain the same.

Back to the child on the swing. How do we keep the swing going (oscillating)? Well we have to keep giving it energy. When do you give it energy? Well if you are a good swing pusher, you know to give the extra small push to sustain the motion of the swing when it is

at the top of its cycle (about to change direction). You do not have to push hard to sustain the swing's oscillations. Once the swing is going you only need to keep supplying energy to overcome the losses in the swing system. The losses in the swing, which slow it down, are friction.

Everything we have just discussed about resonance in mechanical systems (swings, guitars and the like) are equally applicable to electronic resonant circuits.

LC RESONANT CIRCUITS

Not every LC (inductive and capacitive) circuit is resonant. A resonant circuit is only resonant if we use it as a resonant circuit. Though every LC circuit will have a resonant frequency, we don't have to use it as a resonant circuit. We are going to talk about LC circuits that we want to resonate.

If we connect a capacitor and an inductor in parallel as shown in figure 5, nothing happens, as there is no energy in the circuit. This is just like the child's swing before we push it.

Now suppose we give the parallel LC circuit some energy. There are many ways we can give this circuit a pulse of energy. One way would be to expose the inductor to a moving magnetic field (Faradays Law of induction). We could disconnect the capacitor, charge it then connect it back to the inductor.

The latter is what I have done in figure 5. I have charged the capacitor and then connected it back in parallel with the inductor.
WHAT HAPPENS?

The capacitor will begin to discharge through the inductor. As it does so, a magnetic field will be created around the inductor. Eventually the capacitor will be discharged.

Now the energy has moved from the electric field of the capacitor into the magnetic field of the inductor. When the capacitor stops creating current in the circuit the magnetic field around the inductor cannot stay there, as it has no current to support it.

The magnetic field about the inductor will now begin to collapse into the inductor. In doing so there will be relative motion between the inductor and the magnetic field and an emf will be induced into the inductor.

This emf will cause a current to flow in the opposite direction and will cause the capacitor to charge. Eventually all the energy from the magnetic field will have returned to the circuit and the capacitor will be charged (storing energy in its electric field).

The capacitor will begin to discharge through the inductor again and so on.

This circuit is oscillating. The shape of the voltage produced is a sine wave. These oscillations would go on forever except, just like the swing, there are losses in the circuit, which dissipate energy. The losses are the resistance in the wire of the inductor, the

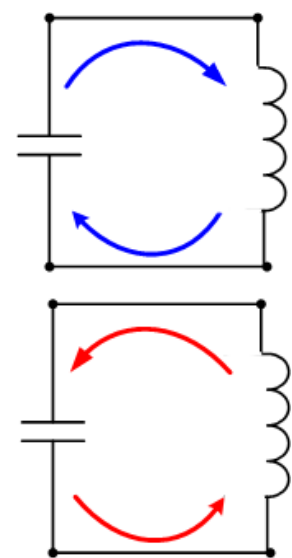


Figure - 5

circuit, and the plates of the capacitor. There is also a little energy lost in the dielectric of the capacitor. The oscillation of this circuit then, is a damped oscillation.

If we wanted a continuous oscillation, we would have to supply the circuit above with energy to overcome the losses. Such a circuit is called an oscillator.

RESONANT FREQUENCY

If you thought about the circuit we have just described, you could probably guess what determines the resonant frequency or frequency of oscillation. Imagine if the capacitor was huge and took ages to discharge through the inductor. Likewise if the inductor is large. You would expect the oscillations or resonant frequency to be low. You would be right. Large values of L and C resonate at a low frequency whereas small values of L and C resonate at higher frequency.

To work out the exact resonant frequency (f_r) of a tuned circuit we can use the following 'must know' equation:

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

The resonant frequency of a tuned circuit is inversely proportional to the square root of the product of the inductance and capacitance. The equation gives the resonant frequency in Hertz, and L and C must be entered in Henrys and Farads respectively.

You will not have to use this equation in the exam, however you may be asked to identify the equation from amongst others. You must know how to increase or decrease the resonant frequency of a tuned circuit.

The same equation is also used to find the resonant frequency of a series LC circuit.

Example.

A tuned circuit (series or parallel) consists of an inductor of 100 microhenries and a capacitor of 250 picofarads. On what frequency will it resonate?

$$100 \text{ microhenries} = 100 \times 10^{-6} \text{ Henrys}$$

$$250 \text{ picofarads} = 250 \times 10^{-12} \text{ Farads}$$

Inserting these values into the equation we get:

$$\text{Resonant frequency } (f_r) = 1 / (2 \times \pi \times \text{squareroot } (100 \times 10^{-6} \times 250 \times 10^{-12}))$$

$$\text{Resonant frequency} = 1.007 \text{ megahertz.}$$

It was not by accident that I chose the values of L and C used in this example. The resonant frequency is close to 1 MHz, which is roughly the centre of the AM radio broadcast band. You would need approximately these values of L and C to make a broadcast band radio receiver or a 'crystal set'.

THE 'Q' OF A RESONANT CIRCUIT

The 'Q' is a term used for the 'quality' of a tuned circuit. Generally, we want as high a Q as possible. The more losses a tuned circuit has the lower the Q. The less losses a tuned circuit has the higher the Q. There are equations for calculating the Q of a tuned circuit, and for that matter for capacitors and inductors. However, you just really need to know what Q is.

IMPEDANCE OF TUNED CIRCUITS

When you read 'tuned circuits' you can also think of resonant circuits, as we are discussing one and the same thing. Parallel and series tuned circuits have different properties which allow us to use them in electronics for different purposes.

You must remember this:

If there were no losses in tuned circuits (never possible) the following would be true:

Parallel resonant circuits have infinite impedance.
Series resonant circuits have zero impedance.

In the real world though resonant circuits have some losses, and so we have to say:

Parallel resonant circuits have maximum impedance at resonance.
Series resonant circuits have minimum impedance at resonance.

The impedance **TYPE** for both series and parallel LC circuits at resonance is **RESISTIVE**. The impedance of a series resonant LC circuit is minimum and resistive. The impedance of a parallel LC circuit at resonance is maximum and resistive.

There is no net reactance at resonance since $X_L = X_C$ – this is why we say that resonant circuits are resistive.

IMPEDANCE TYPE ABOVE AND BELOW RESONANCE

Now this topic is not an exam requirement. However it is very useful for all manner of things to understand what the impedance type is of a resonant circuit above and below resonance. This is very useful when it comes to tuning antennas.

First what do we mean by impedance type?

Well, impedance is made up of three oppositions:

1. Resistance (not effected by frequency).
2. Inductive reactance – increases with frequency.
3. Capacitive reactance – decreases with frequency.

All of these oppositions are measured in Ohms. All oppose current flow. The total combined effect on a circuit of all of these oppositions is called impedance, and it too is measured in Ohms.

Now think of a circuit at resonance. At resonance $X_L = X_C$. So at resonance the only opposition is resistance, or a better way of putting it, the impedance type of a resonant circuit is resistive. The impedance type of both series and parallel circuits at resonance is resistive because $X_L = X_C$. Anything which is resonant has no reactance, only resistance.

IMPEDANCE TYPE OF A SERIES LC CIRCUIT – OFF RESONANCE

An easy way to work out what the impedance type of a series circuit used above or below its resonant frequency, is to think, which reactance (L or C), has the largest voltage drop. Above resonance X_C will decrease and X_L will increase, therefore X_L has the largest voltage drop and the impedance type is inductive. Below resonance the X_C is greater than X_L . The capacitance will have the greater voltage drop, and the circuits impedance type is capacitive.

IMPEDANCE TYPE OF A PARALLEL LC CIRCUIT – OFF RESONANCE

With parallel LC circuits the largest branch current determines the impedance type. Say to yourself, which branch will have the least reactance and therefore draw the most current. Above resonance X_L increases and X_C decreases, and the capacitive branch will draw the most current. Above resonance a parallel circuit is capacitive. Below resonance then, it has to be inductive.

SUMMARY – IMPEDANCE TYPE

Parallel LC circuits:

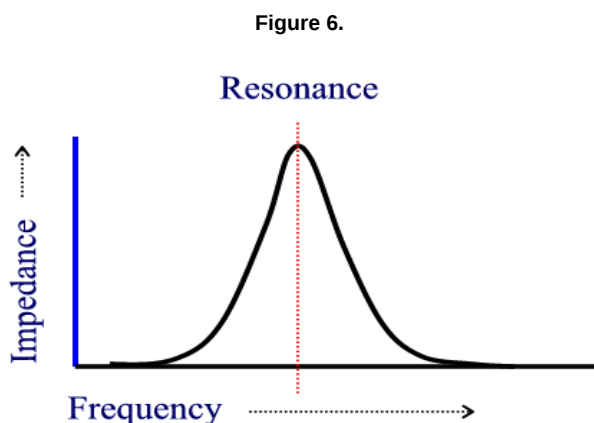
1. Above their resonant frequency are capacitive.
2. Below their resonant frequency are inductive.

Series LC circuits:

1. Above resonance are inductive.
2. Below resonance are capacitive.

Knowing this and the logic behind it is not required for the exam. This knowledge is very useful when experiment with tuned circuits and antenna tuning.

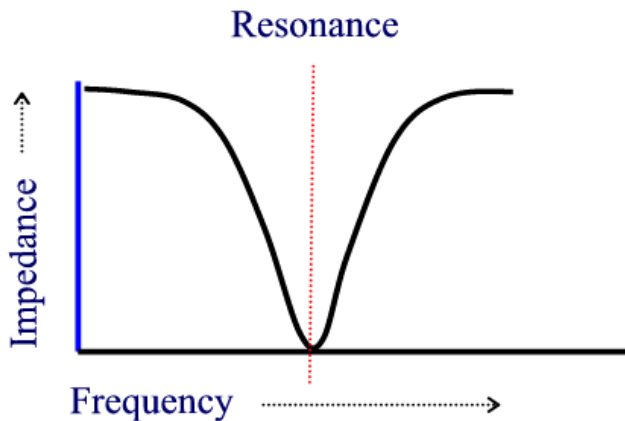
IMPEDANCE OF LC CIRCUITS VERSUS FREQUENCY



A parallel LC circuit has maximum impedance at resonance. At resonance the impedance is maximum and resistive. Either side of resonance the impedance will be less. An LC circuit with no losses has infinite impedance at resonance. If there were a resistance in parallel with a parallel resonant circuit then the impedance of the circuit would be equal to the parallel 'R'. For example, an infinite impedance (which is the same as an open

circuit) in parallel with a 100 K Ω resistor would give a total impedance at resonance of 100 K Ω .

Figure 7.



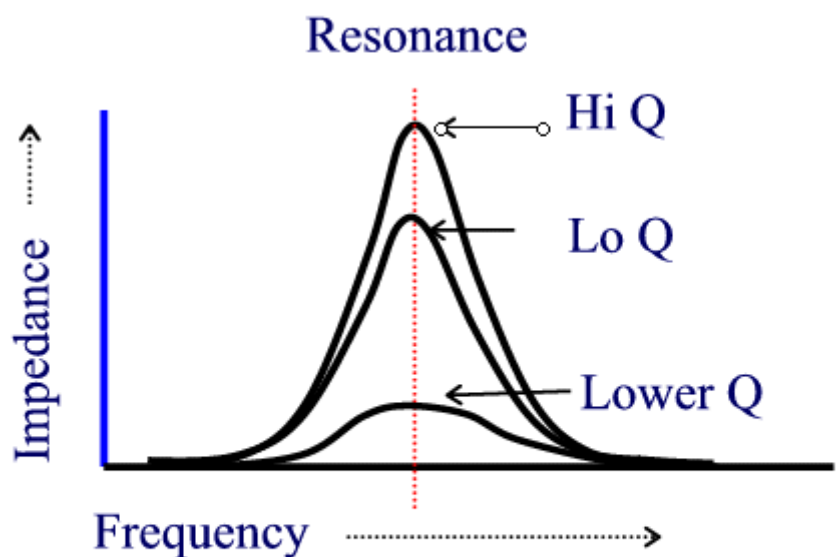
Series LC circuit - At resonance impedance is Minimum

A series LC circuit has minimum impedance at resonance. At resonance the impedance is minimum and resistive. Either side of resonance the impedance will be greater. A series LC circuit with no losses has zero impedance at resonance. If there were a resistance in series with a series resonant circuit then the impedance of the circuit would be equal to the series 'R'. For example a zero impedance (which is the same as a short circuit) in series with a 100 K Ω resistor would give a total impedance at resonance of 100 K Ω .

THE EFFECT OF LOSSES ON THE 'Q' OF TUNED CIRCUITS

We know that in the real world we cannot have parallel or series tuned circuits with no losses at all. The effect of losses on a tuned circuit is to **flatten out** the impedance curves shown. The higher the Q the sharper the curve.

Tuned circuits can be used to make filters and select radio stations. Sometimes we want the curve (often referred to as the selectivity curve) to be sharp (a high 'Q'). Other times we want it to be flat and not as selective so we may actually add losses in the form of resistance to flatten the curve out.



High Q gives a sharp curve
Low Q gives a flatter curve

Figure 8 shows the curves of a parallel tuned circuit with three values of 'Q'. For a series resonant circuit the graph would be inverted.

Figure 8.

'Q' OF A LONE INDUCTOR OR CAPACITOR

This is just a reminder that tuned circuits are not the only thing that have a 'Q' (quality factor). A perfect (also called pure) inductor has no losses and therefore does not dissipate any power. Such an inductor would have an infinite 'Q'. The same applies to a capacitor with no losses, its 'Q' would be infinite and it also would not dissipate any power.

In reality an inductor has losses due to the resistance in its winding and some core losses, hysteresis loss and perhaps some eddy currents if it is a choke. A capacitor has mostly dielectric loss and some loss due to the resistance in its plates.

The losses in an inductor or capacitor can be represented by a resistor in series with them. The equation for finding the 'Q' of a lone inductor or capacitor is simply to divide the reactance by the resistance. **You must remember these two equations.**

'Q' of a lone inductor or capacitor:

$$Q = X_L / R \quad \text{or} \quad Q = X_C / R$$

Adding a resistance to a capacitor, inductor or any resonant circuit reduces its Q because resistance is the only electrical property that dissipates heat.

FLYWHEEL EFFECT



A steamroller has a Flywheel – smaller wheel shown in red.

What is a flywheel? In mechanics, a flywheel is a wheel with a large mass (weight). If you get the flywheel turning it will continue to turn for a long time because of its mass. A flywheel is a means of storing energy and releasing it later.

The science fiction writer Isaac Asimov predicted that one day electric power stations would use gigantic flywheels weighing hundreds of thousands of tonnes. The problem power stations have is that the demand on the generators varies enormously and you cannot, particularly with fossil fuel stations, turn the furnaces up and down at a whim. Imagine though if you had a gigantic flywheel, perhaps floating on mercury or a magnetic field to avoid friction. All of the energy created by the power station is used to make the gigantic flywheel move. You would be storing an enormous amount of energy in this flywheel. It is so massive that it would continue to rotate for months left untouched. When we wanted energy from the flywheel we could make contact with it and have it drive a generator for us. So, even though the output of the power station may be irregular, the power available from the flywheel would be regular because of the massive amount of stored energy.

Now a resonant circuit exhibits the same properties of a flywheel. This effect of resonant circuits is called the flywheel effect. This is how it works.

Suppose we want a sine wave to be produced. We know that a resonant circuit will produce a sine wave if it is given energy. We know that the sine wave output from a resonant circuit will dampen if we don't keep on providing it with energy.

Suppose we have pulses of energy, very short pulses, and suppose we have 1 million of them per second. Let's use these pulses to start a resonant circuit oscillating. Let's make the resonant frequency of the tuned circuit 3 megahertz. This would mean that the resonant circuit would get a pulse of energy and produce 3 cycles of sine wave output. Now a tuned circuit of reasonable Q will be able to run fine for 3 cycles. Then our next pulse comes along, and away the tuned circuit goes again for another three cycles. The output of the resonant circuit is a sine wave at 3MHz. The input is some sort of 'cruddy' pulses at 1MHz.

So, the resonant circuit is producing a 'flywheel effect' for us.

There are many situations in electronics where we want to convert pulses to sine waves or double or triple the frequency of a signal. One way of doing this is with the flywheel effect of a resonant circuit.

If you have trouble visualising the flywheel effect of a resonant circuit think back to the child on a swing (child not required). You pulse the swing with energy every time you push it. You can let the swing go say for three cycles then pulse (push) it again. The swing is converting pulses of energy from you into a smooth motion at three times the frequency (in this example). The frequency of your pulses (pushes) is 1/3 the frequency of the swing.

APPLICATIONS USING RESONANT OR TUNED CIRCUITS

There are hundreds of applications for tuned circuits and it will all come together as we continue the course, however I feel compelled to give you a preview.

An application using a parallel resonant circuit:

The diagram of figure 9 shows an antenna connected to a parallel tuned circuit that is in turn connected to the ground.

Now, remember a parallel tuned circuit has very high impedance at one frequency only, its resonant frequency.

All of the radio waves that pass by the antenna will induce a very small voltage into the antenna. Let's say our resonant circuit is tuned to 1 MHz. An AM radio station on 1.5 MHz will cause a radio wave to pass by the antenna and induce a voltage into it. This voltage will cause a current to flow down the antenna cable and through the tuned circuit to earth. At 1.5 MHz the inductor and capacitor are not resonant and the impedance of the parallel circuit will be very low. $E=IR$ or if you like $E=IZ$ (Z =impedance). The voltage created across the LC parallel circuit will be very low, as its impedance is very low. The same story will go for all other radio signals that induce a voltage into the antenna EXCEPT 1Mhz. At 1MHz the LC circuit is parallel resonant and will be very high impedance. The small current through the parallel resonant circuit will produce a significant voltage across it compared to all the other radio signals.

An output voltage will appear at the output terminals of the signal that the parallel tuned circuit is tuned to. So here we have the basic method of selecting the desired radio signal from the many that are present at the antenna.

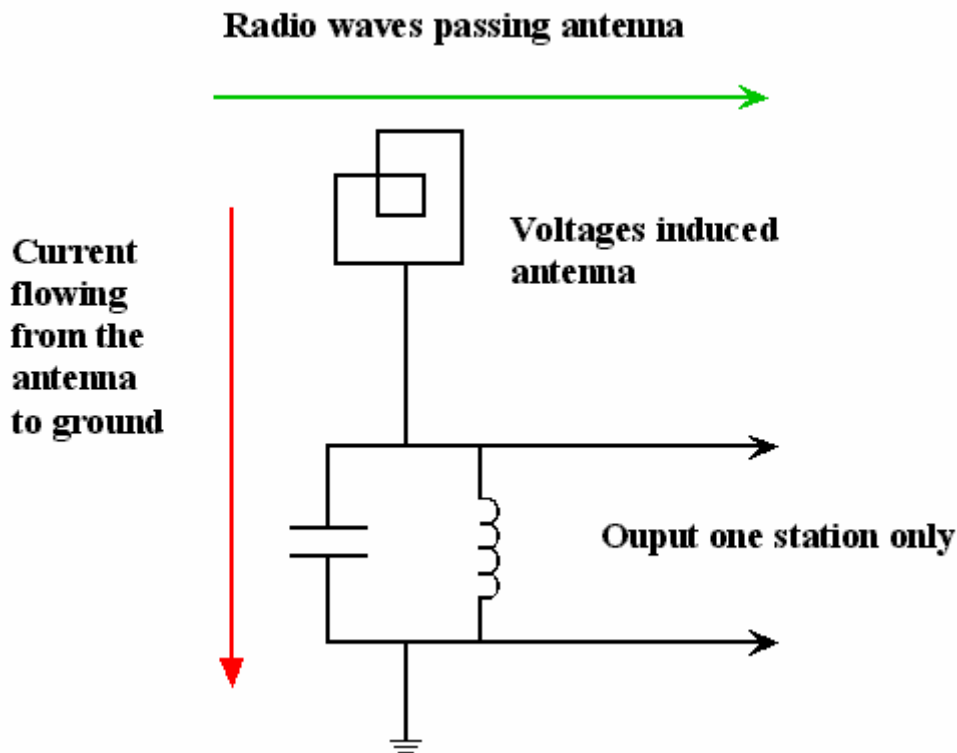


Figure 9.

An application for a series resonant circuit:

Recall that at resonance a series tuned circuit has extremely low impedance. If you like you can think of it as zero impedance. Zero impedance is like a piece of conductor. If you place a conductor across the back of your TV on the terminals where the signal comes into the set, do you think you would get much of a TV picture?

I hope you said no. A short circuit on a TV antenna would stop all signals from entering the TV set and you would get squat!

A series resonant circuit has zero (extremely low) impedance at one frequency only, its resonant frequency.

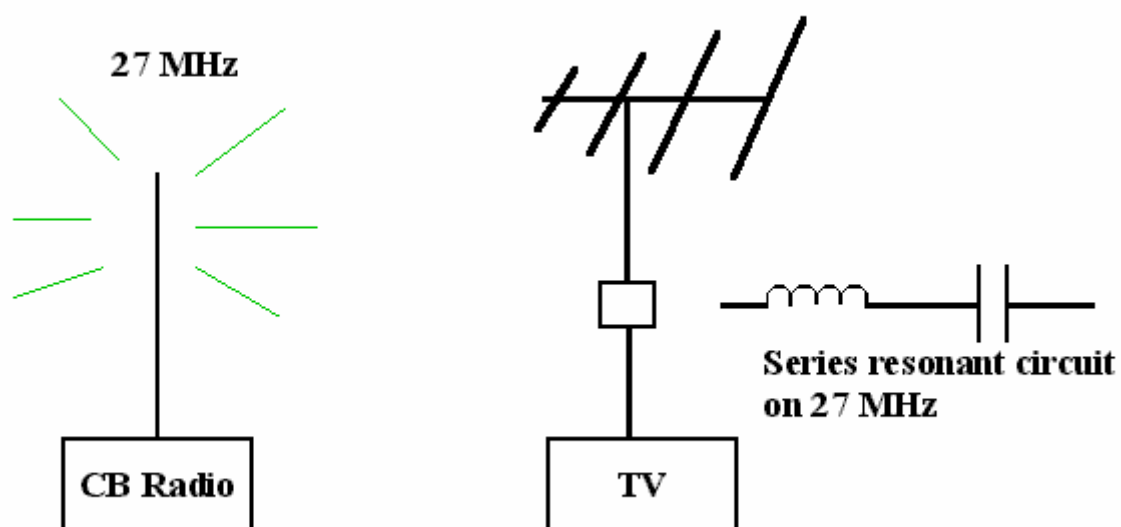
In figure 10 we have an interference problem from a 27 MHz CB radio getting into a nearby television set. This type of interference is called receiver overload. The problem is that the TV can't reject the 27MHz CB signal, as it is so strong and so close.

We have placed a filter in the box shown on the TV cable. In that box there is a series LC circuit which is resonant on 27 MHz.

A 27MHz signal trying to get to the TV will 'see' a short circuit and will be stopped. However all other signals will be unaffected.

The interference is stopped and the TV owner can enjoy watching television.

Figure 10.



A fine word on Q and something to remember because it seems to puzzle many -- in radio circuits:

Low Q means broad bandwidth or poorer selectivity
High Q means narrow bandwidth or better selectivity

End of Reading 19.

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